3.7 Classical control, pseudo-continuous method

3.7.1 Generalities

In this section, the approximations introduced in Section 3.6 will be applied for the classical control through the pseudo-continuous method.

Initially, the equivalent transfer function of a standard controller will be determined. Therefore study the basic relations on the level of the pseudo-continuous control circuit will be studied and a resulting small time constant introduced. Afterwards, the variable and symmetric optimum criteria will be resumed, which permit a simple but efficient dimensioning of the standard controller coefficients. Finally, the particularities of the cascaded control will be mentioned.

For the choice and dimensioning of the controllers, the limitation of the controller output will be discarded, assuming sufficiently small variations. Subsequently, the correction measures of the controller dynamic behaviour should be considered when the limitations take place.

In relation with the sampled method, a significant simplification is achieved whilst applying the methods of a continuous control. These methods are presented in an exhaustive manner in [1], Chapter 10.

3.7.2 Equivalent transfer function of a standard controller

The established relations in Section 3.6.14 allow the deduction of the *equivalent transfer function* for the pseudo-continuous method. As an example, the process for a *PID controller* will be shown ([1], § 9.5.4).

The output signal $y_R[k]$ of a *digital PID controller* is given by the components P, I and D. With a control error of $e[k] = w[k] - y[k]$, one obtains

$$y_R[k] = K_p e[k] + K_i \sum_{i=0}^{k} e[i] + K_d \left(e[k] - e[k-1]\right) \tag{3.119}$$

It is noted that the *output signal* y_R *of the controller* forms either the control signal u_{cm}, or the setpoint w_a in the case of the cascaded control.

With the help of the relations (3.112) and (3.116), an approximation can be made

$$y_R(s) \cong K_p e(s) + K_i \frac{1 + sT_E/2}{sT_E} e(s) + K_d \frac{sT_E}{1 + sT_E/2} e(s) \tag{3.120}$$

from which the equivalent transfer function becomes evident.

$$G_{\mathrm{Re}}(s) = \frac{y_R(s)}{e(s)} = K_p + \frac{1 + sT_E/2}{sT_E} K_i + \frac{sT_E}{1 + sT_E/2} K_d = \\ = \frac{1 + s\dfrac{K_p + K_i}{K_i} T_E + s^2 \dfrac{K_d + K_p/2 + K_i/4}{K_i} T_E^2}{s\dfrac{T_E}{K_i}(1 + sT_E/2)} \tag{3.121}$$

In the present case, the lowercase symbols are reserved for the Laplace transformation since they refer to normalised quantities.

A comparison with the transfer function of a continuous PID controller ([1], § 9.2.4)

$$G_R(s) = \frac{(1 + sT_n)(1 + sT_v)}{sT_i} \tag{3.122}$$

shows that

$$G_{\mathrm{Re}}(s) = \frac{1}{1 + sT_E/2} G_R(s) \tag{3.123}$$

Due to the use of the pseudo-continuous method for a PID controller, an additional small time constant $T_E/2$ appears. This happens thanks to the fact that the D-component is implemented approximately by the difference of values between two sampling instants.

Table 3.4 contains the expressions for the equivalent transfer functions $G_{Re}(s)$ of different standard controllers. It is noted that the *PI controller* does not possess the small time constant $T_E/2$. The *I controller* presents a particularity: the small time constant of $T_E/2$ appears here on the numerator forming a zero in $G_{Re}(s)$.

Table 3.4 Equivalent transfer function $G_{Re}(s)$ and relations between the digital controllers and continuous controllers.

Controller	$G_{Re}(s)$	K_i	K_p	K_d
I	$\frac{1+sT_E/2}{sT_i}$	$\frac{T_E}{T_i}$	-	-
PI	$\frac{1+sT_n}{sT_i}$	$\frac{T_E}{T_i}$	$\frac{T_n-T_E/2}{T_i}$	-
PID	$\frac{(1+sT_n)(1+sT_v)}{sT_i(1+sT_E/2)}$	$\frac{T_E}{T_i}$	$\frac{T_n+T_v-T_E}{T_i}$	$\frac{(T_n-T_E/2)(T_v-T_E/2)}{T_iT_E}$

3.7.3 Relations between the coefficients of a digital controller and the time constants of a continuous controller

A comparison between (3.121) on one hand and (3.122), (3.123) on the other hand shows that between the coefficients of a digital controller and a continuous controller, the following relations exist for the PID controller:

$$\left.\begin{aligned} T_i &= \frac{T_E}{K_i} \\ T_E^2\, T_n + T_v &= \frac{K_p + K_i}{K_i} T_E \\ T_n T_v &= \frac{K_d + K_p/2K_i/4}{K_i} \end{aligned}\right\} \tag{3.124}$$

Therefore, from the time constants T_i, T_n and T_v of the continuous controller, the coefficients K_i, K_p and K_d of the digital controller can be calculated as

$$\left.\begin{aligned} K_i &= \frac{T_E}{T_i} \\ K_p &= \frac{T_n + T_v - T_E}{T_i} \\ K_d &= \frac{T_n T_v}{T_i T_E} - \frac{2(T_n + T_v) - T_E}{4T_i} = \frac{(T_n - T_E/2)(T_v - T_E/2)}{T_i T_E} \end{aligned}\right\} \tag{3.125}$$

In Table 3.4, similarly the corresponding relations for the other standard controllers are found.

3.7.4 Block diagram of the pseudo-continuous control circuit

Fig. 3.50 shows the block diagram of the pseudo-continuous control circuit with the equivalent transfer function $G_{Re}(s)$ of the controller according to Table 3.4. As reminded, the latter could contain a small time constant. The second block with $G_r(s) = 1/(1+sT_r)$ and the third block with $G_{me}(s) = 1/(1+sT_E/2)$ model the delay T_r caused by the processing time of the control algorithm and the hold element (§ 3.6.15). In certain cases, it is possible that there is no hold element. Therefore, $G_{me}(s) = 1$ has to be posed. The action module has been modelled by the block with the transfer function of $G_{cm}(s) = K_{cm}/(1+sT_{cm})$, where T_{cm} is the small time constant. The controlled system has the transfer function $G_s(s)$. Finally, the feedback chain includes the measuring device with the transfer function of $G_{Mes}(s) = 1/(1+sT_{Mes})$. Most often, the time constant T_{Mes} can be discarded by setting $G_{Mes}(s) = 1$. Sometimes, a small time constant needs to be introduced, such as for the digital speed measurement according to Table 3.3 for example. On this level, the final smoothing filter must be taken into account.

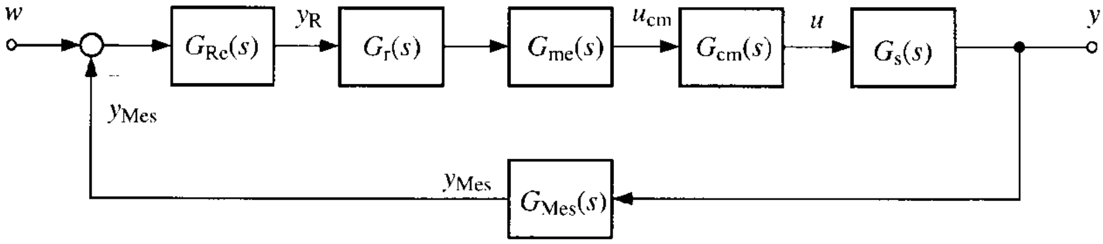

Fig. 3.50 Block diagram of the pseudo-continuous control circuit.

3.7.5 Resulting small time constant

In order to facilitate the pseudo-continuous dimensioning of the digital controllers, it is sensible to gather the small time constants into one, according to the principle of the sum of small time constants ([1], § 3.6.8). It is therefore possible to reduce significantly the block diagram of the control circuit, as shown by Fig 3.51.

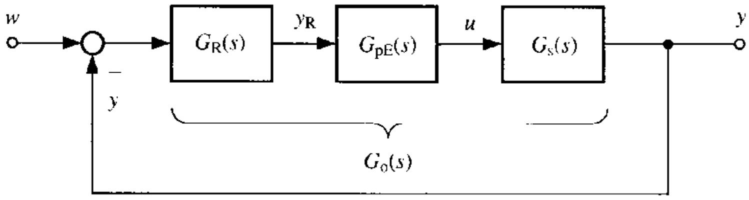

Fig. 3.51 Reduced block diagram of the pseudo-continuous control circuit.

Instead of the action module, only one block appears with a transfer function of

$$G_{pE}(s) = \frac{K_{cm}}{1+sT_{pE}} \tag{3.126}$$

The term K_{cm} is the *transfer factor* of the action module. The *resulting small time constant* is given by

$$T_{pE} = \kappa T_E + T_{cm} + T_r + T_{Mes} \tag{3.127}$$

The factor κ depends on the employed controller type. This factor is indicated in Table 3.5

Table 3.5 Factor *κ* for the calculation of the resulting small time constant T_{pE}.

Controller	I	PI	PID
κ	0	0.5	1

During this transformation, a block needs to be introduced with the transfer function $1/G_{Mes}(s)$ related to the setpoint *w* ([1], § 10.2.3). If the time constant T_{Mes} is small, this block can be omitted.

3.7.6 Choice and pseudo-continuous dimensioning of standard controllers

The choice and pseudo-continuous dimensioning of standard controllers can be done according to the usual dimensioning criteria of standard controllers ([1], Chapter 10). In the following, variable and symmetric optimum criteria will be presented.

With this dimensioning, the time constants T_i, T_n, T_v of the continuous controller are obtained. Through the relations contained in Table 3.4, the coefficients K_i, K_p, K_d of the digital controller are found. These coefficients have to be introduced into the control algorithm.

3.7.7 Dimensioning according to the Magnitude Optimum criterion

For the dimensioning according to the *variable optimum criterion* (criterion on the harmonic response), the most frequent case with hold element is to be considered. A controlled system of order $n_s = 2$ is assumed with two *dominant time constants* T_1 and T_2. Its transfer function is

$$G_s(s) = \frac{K_s}{(1+sT_1)(1+sT_2)} \tag{3.128}$$

where K_s is the *transfer factor*. By setting $T_2 = 0$ or also $T_1 = 0$, a controlled system of order $n_s = 1$ or 0 is obtained.

The Magnitude Optimum criterion ([1], Sect. 10.3) for the choice and dimensioning of a standard controller requires the following steps:

- according to the order of the controlled system $n_s = 2$, 1 or 0, a PID, PI or I controller must be chosen (the integral component is needed to cancel the steady state control error);
- then, the resulting small time constant T_{pE} can be determined with the aid of (3.127);
- with the controller's time constants T_n and T_v, the time constants of the controlled systems are compensated

  $$T_n = T_1 \ ; \quad T_v = T_2 \tag{3.129}$$

  while supposing that $T_1 \geq T_2$ (it is not admissible to compensate a small time constant ([1], § 10.3.4);
- the controller's integration time constant is then expressed as
-

  $$T_i = 2KT_{pE} \tag{3.130}$$

  where

  $$K = K_{cm}K_s \tag{3.131}$$

  is the resulting transfer factor.

Also, the dimensioning is very simple. The rules for the three controllers I, PI and PID are resumed in Table 3.6.

Table 3.6 Choice and dimensioning of the standard controllers according to the variable optimum criterion.

n_s	Controller	T_n	T_v	T_i
0	I	-	-	$2KT_{pE}$
1	PI	T_1	-	$2KT_{pE}$
2	PID	T_1	T_2	$2KT_{pE}$

It has to be noted that the standard controllers are not well adapted for the higher order systems ([1], Section 10.3). In practice, this does not mean a too strict restriction as most often the cascaded control is applied when the controlled subsystems have an order of $n_s = 1$ or 2. Moreover, the standard controllers are poorly adapted to control the oscillating systems.

3.7.8 Symmetrical optimum based dimensioning

The dimensioning, based on the Magnitude Optimum criterion and discussed in the previous section, yields a very good dynamic behaviour while subjected to setpoint variations. The dynamic behaviour related to the disturbance variable is acceptable when the dominating time constants are not too big as compared to the small time constant ([1], Section 10.3).

If, on the contrary, a rapid reaction to the disturbance value variations is demanded, especially when the dominating time constants are big, the controller should be dimensioned according to the symmetrical optimum (See Appendix A1.2). If the controlled system shows an integral behaviour, this dimensioning criterion is the only possible.

In this context, the topic is limited to the most important case in the power electronics domain, namely the first order controlled system with integral behaviour. Its transfer function is then given by

$$G_s(s) = \frac{1}{sT_I} \tag{3.132}$$

where TI is the integration time constant. Moreover, the presence of a hold element is supposed.

The symmetrical optimum criterion (A1.2) for the selection and dimensioning of a standard controller requires the following procedure:

- while the system order is $n_s = 1$, the PI controller must be selected (even if there is an integral behaviour in the controlled system, it is inevitable to choose a controller with integral component in order to mitigate the influence of the disturbance variable in the steady state ;
- then, the resulting small time constant T_{pE} can be determined with the help of (3.127);
- the controller's time constant T_n is to be adapted to T_{pE} according to

$$T_n = 4T_{pE} \tag{3.133}$$

- the controller's integration time constant is obtained by

$$T_i = 8\frac{T_{pE^2}}{T_I}K_{cm} \tag{3.134}$$

- it is necessary to add a corrector for the setpoint variable with the time constant

$$T_{cw} = T_n = 4T_{pE} \tag{3.135}$$

As it is explained in Appendix A1, a corrector must be added for the setpoint variable, also known as the setpoint filter. Actually, with the symmetrical optimum based dimensioning, a very good dynamic behaviour is obtained in regard of the disturbance variable. However, a large overshoot appears during the set-point variations (A1.2.4). This overshoot can be strongly reduced with the aid of the setpoint filter with the transfer function

$$G_{cw}(s)=\frac{w'(s)}{w(s)}=\frac{1}{1+sT_{cw}} \tag{3.136}$$

At the controller's input, the setpoint w' is applied.

3.7.9 Conditions allowing the use of pseudo-continuous method

The pseudo-continuous method can be applied for the standard controllers if the sampling interval satisfies the condition

$$T_E \le T_{ns}/2 \tag{3.137}$$

where T_{ns} is the smallest of the dominating time constants. Sometimes it is possible to admit $T_E < T_{ns}$. The small time constant $T_p = T_{cm} + T_M$, introduced by the control and measuring device must respond to the condition $T_p < T_{ns}/4$.

If the controlled system does not have any dominating time constant (proportional behaviour), necessitating the application of an I controller, the sampling interval must respect the condition

$$T_E \le T_p \tag{3.138}$$

It is generally more restrictive (in terms of T_p) than the condition (3.137).

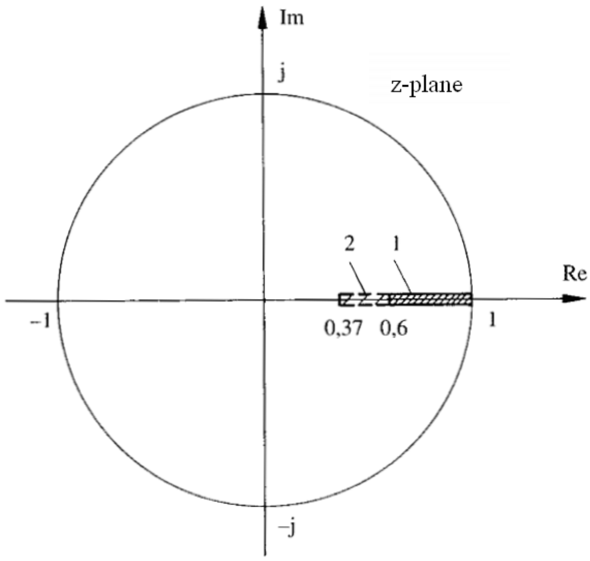

Fig. 3.52 The domain of the poles on the z-plane for a pseudo-continuous processing of the classical control; 1: $z = e^{-T_E/T_k}$ for the dominating time constants; 2: $z = e^{-T_E/T_p}$ for a proportional behaviour of the controlled system.

Fig. 3.52 shows the domain of the poles on the z-plane allowing the use of the pseudo-continuous method. The domain is practically limited to the real poles. Effectively, the standard controllers are not well adopted to control a poorly dampened oscillating system.

Many comparative calculations between the sampled and pseudo-continuous dimensioning have shown the validity of the pseudo-continuous method, in terms of either the controller coefficient values or the trajectory of transient phenomena.